Name:____________________ Math 260
Start Time:________________ Quiz 9 (25 min)
End Time:_________________
Date:_____________________

1. (2 points) Are the vectors $\begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 6 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} -16 & 16 \\ 10 & 0 \end{bmatrix}$ linearly independent? Why or why not? If not, write the zero vector as a linear combination of these 3 vectors where not all coefficients are 0.

Suppose scalars $c_1, c_2, c_3 \in \mathbb{R}$ exist such that

$$c_1 \begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} + c_2 \begin{bmatrix} -3 & 6 \\ 2 & 0 \end{bmatrix} + c_3 \begin{bmatrix} -16 & 16 \\ 10 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

$$\left[\begin{array}{ccc|c} 2 & -3 & -16 & 0 \\ 4 & 6 & 16 & 0 \\ -1 & 2 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{RREF} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$c_3 = t$

$c_2 + 4c_3 = 0 \Rightarrow c_2 = -4t$

$c_1 - 2c_3 = 0 \Rightarrow c_1 = 2t$

So there are many solutions (bec. any value of t produces a different solution).

Since $c_1 = c_2 = c_3 = 0$ is not forced on us, these vectors are $\boxed{\underline{\text{NOT}} \text{ linearly independent.}}$

Let $t=1 \Rightarrow c_1 = 2 \quad c_2 = -4$ and $c_3 = 1$

$$\Rightarrow \boxed{2\begin{bmatrix} 2 & 4 \\ -1 & 0 \end{bmatrix} - 4\begin{bmatrix} -3 & 6 \\ 2 & 0 \end{bmatrix} + 1\begin{bmatrix} -16 & 16 \\ 10 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$$

is a linear combination of the given 3 matrices that produces the zero matrix but the coefficients are not all zero.

2. (2 points) Are the vectors $f(x) = e^x$ and $g(x) = x^2$ linearly independent as vectors in $F(-\infty, \infty)$? Why or why not?

Suppose scalars $c_1, c_2 \in \mathbb{R}$ exist such that

$$c_1 e^x + c_2 x^2 = 0$$

plug in $x=0$ $\Rightarrow$ $c_1 e^0 + c_2 (0)^2 = 0$

$\Rightarrow$ $c_1 (1) + c_2 (0) = 0$

$\Rightarrow$ $c_1 = 0$

plug in $x=1$ $\Rightarrow$ $c_1 e^1 + c_2 (1)^2 = 0$

$\Rightarrow$ $0 \cdot e + c_2 (1) = 0$

$\Rightarrow$ $c_2 = 0$

$c_1 = c_2 = 0$ is forced on us.

So $F(x) = e^x$ and $g(x) = x^2$ are linearly independent functions (in $F(-\infty, \infty)$).

3. (3 points) Show that the vectors $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}$ form are both linearly independent and span $\mathbb{R}^3$. (Hint: Use a theorem)

Let $A = \begin{bmatrix}1 & 4 & 0\\2 & 5 & 1\\3 & 6 & 0\end{bmatrix}$. Since $|A| = 6 \neq 0 \Rightarrow A$ is invertible.

By a theorem, since A is invertible, the columns of A are both linearly independent and span $\mathbb{R}^3$.

4. (3 points) Find a basis for $\mathbb{R}^3$ containing the vector $\begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$

Since $\dim(\mathbb{R}^3) = 3$, we need 3 vectors.
I'll just add in vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ (randomly chosen).

Let $A = \begin{bmatrix} 5 & 1 & 0 \\ 0 & 2 & 0 \\ -2 & 3 & 1 \end{bmatrix}$. $|A| = 10 \neq 0$
$\Rightarrow$ A is invertible.

By a theorem, since A is invertible, the columns of A are linearly independent and span $\mathbb{R}^3$.

So $B = \left\{ \begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$ containing $\begin{bmatrix} 5 \\ 0 \\ -2 \end{bmatrix}$.